ROBUST VAGUENESS AND THE FORCED-MARCH SORITES PARADOX

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The ancients attributed both the sorites paradox and the liar paradox to Eubulides, a member of the Megarian school and a contemporary of Aristotle. Yet even though the two paradoxes have been known equally long, the liar has received far more attention than the sorites in philosophy and in logic, both historically and in our own time. The sorites, like Rodney Dangerfield, gets no respect—not much, anyway.

Historically, this neglect perhaps has something to do with the tendency to treat Euclidean mathematics, that great paradigm of precision, as a model for human knowledge generally. This tradition still exerts a powerful influence today, largely via the legacy of logical positivism. And in recent times, the relative neglect of the sorites paradox probably also reflects the recently widespread assumption that interesting human concepts have precise necessary and sufficient conditions, articulable via “conceptual analysis.”

But whatever the reasons might be why the sorites paradox has been neglected and underappreciated, it is time for that to change. Indeed, I think this change is probably inevitable, now that philosophers are widely coming to recognize that most human concepts are not susceptible to the kind of conceptual analysis envisioned in the heyday of High Church analytic philosophy. As we move into the post-analytic era, we must confront the fact that concepts are usually vague. Wherever there is vagueness, there looms the sorites.

My primary purpose in this paper is to urge a new philosophical respect for the sorites paradox. In my view the paradox is much more difficult, much more philosophically deep, and much more fraught with import for metaphysics, semantics, and logic than is generally appreciated. I will explain why I think so, and I will advance some specific positive proposals along the way.

In first the half of the paper (Sections 1-3), I take for granted that vagueness is a genuine and intelligible phenomenon, and that a proper understanding of it
will reveal a satisfactory way to block paradoxical sorites arguments. In Section 1 I distinguish between two notions of vagueness that I call, respectively, robust and wimpy; I argue that genuine vagueness is robust rather than wimpy; and I argue that standard treatments of the sorites paradox are inadequate because they construe vagueness as wimpy rather than robust. In Section 2 I sketch a non-classical logic that reflects the robustness of genuine vagueness and also blocks standard forms of the sorites paradox; I argue that truth itself is vague, and thus that the same logic should hold in a metalanguage for an object language governed by this logic. In Section 3 I give a Tarski-style truth characterization for a simple artificial language employing vague predicates; this truth characterization turns out to have only limited utility in understanding the logic of vagueness, because its implications for logic depend crucially on the logic of the metalanguage itself.

In the second half (Sections 4-7), I plunge into deeper philosophical water. The discussion has the following dialectical structure:

Thesis: Vagueness is impossible.
Antithesis: Vagueness is actual (and hence possible).
Synthesis: Roughly, one kind of vagueness is impossible and another kind is actual.

In Section 4 I set forth the case for the Thesis. The heart of the argument is a form of sorites reasoning that does not get directly addressed by the logic of vagueness sketched in Section 2; I call it the forced march sorites paradox. This version of the paradox, it would seem, reveals that vagueness is just impossible—and thereby also seems to show that the non-classical logic proposed earlier is incoherent. In Section 5 I reverse direction, and set forth the case for the Antithesis. Here the central argument is the overwhelming implausibility that there are any “hidden facts” that somehow yield perfectly precise semantic or metaphysical boundaries wherever we would normally think there is vagueness. In Section 6 I sketch a proposed Synthesis of these two seemingly incompatible positions; the non-classical logic sketched in Section 2, now regarded in a quite different light, plays an important role. In Section 7 I discuss some philosophical implications of the Synthesis.

1. The Robustness of Vagueness.4

I begin with examples of two kinds of paradoxical sorites argument; I will call these, respectively, the quantificational sorites and the conditional sorites. Let ‘Bn’ abbreviate ‘a person with n hairs on his head is bald’. The quantificational sorites argument, for baldness, is this:
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(Q) (1) (n)(Bn ⊃ Bn+1)
    (2) B(0)
:: (3) B(107)

The associated conditional sorites is non-quantificational, and instead employs as premises a huge number of specific conditionals that all follow from the first premise of argument (Q) by universal instantiation:

(C) (1) B(0)
    (2) B(0) ⊃ B(1)
    (3) B(1) ⊃ B(2)
    ...
    ...
    (107+2) B(107-1) ⊃ B(107)
:: (107+3) B(107)

An adequate account of vagueness must, of course, block both forms of sorites argument.

Most recent treatments of vagueness in philosophy and in logical theory draw upon central notions in formal semantics, while also complicating these notions or attempting to generalize upon them. For instance, one approach involves the suggestion that truth and reference come in degrees, rather than being all-or-none notions. Typically this idea is implemented by introducing as truth values all real numbers in the interval from 0 (full-fledged falsity) to 1 (full-fledged truth); and perhaps also construing the numbers in this interval as measures of the degree to which a predicate applies to an object or sequence of objects (0 for full-fledged non-applicability, and 1 for full-fledged applicability). Under this approach, vague predicates do not sharply partition the world into those things that fall under them and those that do not; rather, there is gradual alteration in the degree to which they apply to things, and corresponding gradual alteration in the truth values of associated statements. Take the predicate abbreviated by ‘B’ in arguments (Q) and (C), for instance, and consider the sequence of statements B(0), B(1), ..., B(107). I will call this the baldness sequence.) According to the degrees-of-truth view, there is no number i such that the i-th statement in this statement is true and the (i+1)-th statement is false. Rather, the truth values gradually move downward from 1 to 0, as the number of hairs increases. Thus, many of the premises in argument (C) have a truth value less than one; and so does the first premise in argument (Q). So neither argument is sound.

This approach faces a problem, involving awkward questions like these: Which statement in the sequence is the last whose truth value is 1? Which is the first whose truth value is 0? Prima facie, there just does not seem to be anything about the norms or standards for proper use of the predicate 'is bald' that would
sanction as correct any single specific assignment of precise degrees of truth to the respective statements in the baldness sequence, over against a variety of other candidate-assignments. Additional questions like the following further underscore this point: Between the values of 1 and 0, do truth values change linearly as a function of the number of hairs, or in some other way (e.g., as a sigmoid function—an "S-curve")? If the semantic-value function is nonlinear, which specific nonlinear function is it, and why? These questions lack principled answers, and this fact seems to be a crucial aspect of the vagueness of 'is bald'. It shows that the adoption of any specific assignment of precise degrees of truth, over against any other candidate-assignments that are equally consistent with ordinary usage, would be arbitrary—and hence would be a distortion of the predicate's actual semantic features. This difficulty, which I will call the problem of arbitrary precisification, arises for vague predicates in general. (For any vague predicate there are sequences relevantly analogous to the baldness sequence; I will call these sorites sequences.)

Let an expression E be robustly vague if there is nothing in our actual semantic norms that sanctions any single candidate-precisification of E as correct, over and above various other candidate-precisifications; and let E be wimpily vague if some unique candidate-precisification is so sanctioned, rather than being semantically arbitrary. The import of the preceding remarks is that genuine vagueness is robust, rather than wimpy.7 An adequate semantical treatment of vagueness should respect this robustness, and hence should eschew arbitrary precisification. (Henceforth I will use the terms 'robust' and 'wimpy', and the associated adverbial and nominalized forms, sometimes to talk about vagueness and related phenomena, and sometimes to talk about proposed theoretical accounts of such phenomena.)

The problem of arbitrary precisification arises, in one way or another, for most approaches I know of that attempt to handle vagueness by complicating, or generalizing upon, standard Tarskian or model-theoretic semantics for formalized languages. These proposals prove ultimately wimpy, and thus fail to do justice to the robustness of genuine vagueness. For instance, the problem obviously arises for the proposal to assign so-called "fuzzy sets" (Zadeh 1965) to vague predicates as their extensions; for, these entities are only wimpily fuzzy, not robustly so. For another instance, take the 'supervaluationist' approach. The leading idea is this: since a vague predicate can be assigned any of a variety of equally eligible potential extensions, a statement of the form 'Fa' is true if the referent of 'a' belongs to every eligible candidate-extension of 'F'; is false if the referent of 'a' belongs to no eligible candidate-extension of 'F'; and otherwise is neither true nor false. On the surface, this way of treating vagueness looks laudably robust, since it explicitly acknowledges and accommodates a range of equally good ways to precisify a vague predicate. But its underlying wimpiness becomes evident as soon as one considers its implications for sorites sequences like the baldness sequence: the approach is committed to a sharp dividing point
between the last statement in the sequence that is true and the first statement that is not, and to a sharp dividing point between the last statement that is not false and first that is. But the choice of any specific dividing points is just another form of arbitrary precisification; for, a crucial aspect of the robustness of genuine vagueness is that there is no precise fact of the matter about truth-value transitions in sorites sequences.\textsuperscript{8}

Other recent approaches that attempt to extend standard semantical treatments of formalized languages—e.g., by appealing to an iterative ‘definitely’-operator somewhat analogous to the necessity and possibility operators of modal logic—evidently fare no better. In the end, they all seem committed to some kind of arbitrary precisification in the assignment of truth values to the statements in a sorites sequence, rather than capturing the fact that the distribution of truth values is itself an inherently vague matter. Wimpiness in these accounts is like water in a sealed balloon; squeeze it away at one spot, and it bulges forth somewhere else.

One response I sometimes hear to the charge of wimpiness is this: “Accounts of the kind being criticized are not intended to provide a full theoretical account of vagueness, but rather are simplified, idealized, theoretical accounts—comparable, for instance, to accounts in physics employing assumptions like frictionless surfaces or volumeless point-masses.” But it takes only a moment’s reflection to see the inadequacy of this reply. An assumption like the absence of friction simplifies one’s theoretical account of a physical phenomenon without essentially distorting the phenomenon itself: the actual physical system behaves in a manner that would asymptotically approach the idealized limit-case, were the amount of actual friction to diminish. Furthermore, the same physical theories that apply to idealized systems typically are also applicable to more complex systems in which the idealizations no longer hold—although dropping the idealizations sometimes complicates things in ways that are largely gratuitous for the explanatory or predictive purposes at hand. (One can accommodate friction in physics, when it becomes important to do so.) But the relation between the robustness of genuine vagueness, and the wimpiness attributed to vagueness under standard semantical treatments, is utterly different. For, it is essential to ordinary vagueness that there be no determinate fact of the matter about how the semantic status of the statements in a sorites sequence changes as one proceeds systematically from one end of the sequence to the other. I.e., genuine vagueness is essentially robust. Standard accounts flout this feature, and replace it with one or another kind of artificial precisification that is thoroughly incompatible with it.

Recognizing the robustness of vagueness, however, is only the beginning. The task now is to understand it—i.e., to come to grips with the notion “no determinate fact of the matter about semantic changes in a sorites sequence.” This is anything but easy, as shall be seen.
2. The Logic of Vagueness.

In reflecting on what an adequate overall account of vagueness would have to be like, it is natural to begin by considering matters of logic. What logical principles should govern robustly vague language, and/or reasoning about robustly vague entities (assuming there are such entities)? In this section I address this question, albeit in a partial way. I describe some key features that look like plausible candidates for incorporation into a logic of vagueness.9

It is beyond doubt that the logic we seek must somehow differ from classical logic. For, this is a logical truth in classical logic:

\[(1) \ (n)(Bn \supset Bn+1) \lor (\exists n)(Bn \& \sim Bn+1).\]

Yet the left disjunct of (1),

\[(2) \ (n)(Bn \supset Bn+1),\]

is the major premise for the quantificational sorites argument (Q), whereas the right disjunct of (1),

\[(3) \ (\exists n)(Bn \& \sim Bn+1),\]

asserts the existence of a sharp boundary between the bald and the not-bald.10 Under classical logic, one of the disjuncts (and only one, since (2) and (3) are contradictories) must obtain. So classical logic has to go.

What we seek, then, is some modification of classical logic that both (i) blocks sorites arguments, and (ii) accommodates the robustness of vagueness. To begin with, the intuitively natural thing to say about statements (2) and (3) is that each of them is neither true nor false. Concerning (2), the intuitive reasoning to support this position goes as follows. Suppose that (2) is true. Then by argument (Q), a person with $10^7$ hairs on his head is bald—which is clearly false. Hence, (1) is not true. Now suppose that (2) is false. Then its negation,

\[(4) \sim (n)(Bn \supset Bn+1),\]

is true. So, since (4) is logically equivalent to (3), there is a sharp boundary between the bald and the not-bald. But it's not the case that there is any sharp boundary. Hence, (2) is not false.

The same sort of reductio-style reasoning works for (3). If (3) is true then there is a sharp boundary between the bald and the not-bald; but it's not the case that there is any such boundary; so (3) is not true. If (3) false, then its negation,

\[(5) \sim (\exists n)(Bn \& \sim Bn+1),\]

is true; so by the logical equivalence of (5) and (2) plus argument (Q), a person with $10^7$ hairs on his head is bald—which is clearly false. So (3) is not false.
By analogous reasoning, (1) too is neither true nor false.

So we want statements like (1)-(3) to turn out neither true nor false. And, as illustrated by the preceding reasoning, we want *reductio* arguments to establish not that the *reductio* premise is false (and thus that its classical negation is true), but rather that it is not true.

The reasoning just articulated concerning (1)-(3) was metalinguistic, involving truth and falsity. But it is natural and useful to enrich the object language as well, by adding another form of negation. Let \( \neg \phi \) be true when it’s not the case that \( \phi \) is true; \( \phi \) itself might be false, or might lack truth value altogether. Call this weak negation. Strong negation, by contrast, will work in the manner of negation in classical logic: \( \neg \phi \) will be true when \( \phi \) is false. Although these two forms of negation do not seem to have cleanly distinguishable modes of expression in ordinary language, I do think they both *occur* in ordinary language. So I now stipulate the following usage, to apply henceforth in this paper: ‘it’s not the case that’ is to be understood as the ordinary-language counterpart of \( \neg \), whereas other negation constructions in English will be counterparts of \( \neg \).

At the object-language level of discourse, then, the *reductio* reasoning articulated above establishes the weak negations of (2) and (3). And so, since the strong negation of (2) is equivalent to (3), and the strong negation of (3) is equivalent to (2), that *reductio* reasoning also establishes the weak negations of the strong negations of both (2) and (3). Thus the following are all true:

\[
\begin{align*}
(6) \quad & \neg(n)(Bn \supseteq Bn+1) \\
(I.e., & \text{it’s not the case that for any } n, \text{ if an } n\text{-haired person is bald then an } (n+1)\text{-haired person is bald.}) \\
(7) \quad & \neg(\exists n)(Bn & \& \neg Bn+1). \\
(\text{It’s not the case that there’s some } n & \text{ such that an } n\text{-haired person is bald but an } (n+1)\text{-haired person is not bald.}) \\
(8) \quad & \neg\neg(n)(Bn \supseteq Bn+1). \\
(\text{It’s not the case that not every } n & \text{ is such that if an } n\text{-haired person is bald then an } (n+1)\text{-haired person is bald.}) \\
(9) \quad & \neg(\exists n)(Bn & \& \neg Bn+1).^{11} \\
(\text{It’s not the case that there’s not an } n & \text{ such that an } n\text{-haired person is bald but an } (n+1)\text{-haired person is not bald.})
\end{align*}
\]

As for (1), since both disjuncts are neither true nor false, neither (1) nor its strong negation is true; and so the weak negations of both (1) and its strong negation are true.

Once we have two forms of negation, it also becomes natural to introduce two kinds of conditional to go with them. Just as \( \phi \supset \psi \) is equivalent to \( \neg \phi \vee \psi \), it is natural to have a conditional \( \phi \supset \psi \), equivalent to \( \neg \phi \vee \psi \). Such a conditional is non-vacuously true when the antecedent and consequent are both true; is vacuously true when it’s not the case that the antecedent is true; and is
false when the antecedent is true and the consequent is false. (Henceforth, by stipulation, I will use the ordinary-language locutions ‘φ only if ψ’ and ‘ψ if φ’ for the conditional expressed symbolically by ‘→’, and the ordinary-language locution ‘if φ then ψ’ for ‘⊃’.)

We now have another way besides statement (2) to formulate an induction premise for a sorites argument, and another way besides (3) to formulate a statement asserting the existence of a sharp boundary. The second kind of induction premise is this:

\[(10) \ (n)(B_n \rightarrow B_{n+1}) \]

(For any n, an n-haired person is bald only if an (n+1)-haired person is bald.)

(Equivalent to: For any n, either it’s not the case that an n-haired person is bald, or an (n+1)-haired person is bald.)

This statement is no less potent than (2), as regards the sorites paradox. If we use it in place of (2) in an argument of form (Q), the resulting argument is valid too:

\[(Q2) \ (1) \ (n)[B(n) \rightarrow B(n+1)] \]
\[(2) \ \ B(0) \]
\[\vdots \ (3) \ \ B(10^7) \]

Likewise, the corresponding conditional sorites argument is valid as well:

\[(C2) \ (1) \ \ B(0) \]
\[(2) \ \ B(0) \rightarrow B(1) \]
\[(3) \ \ B(1) \rightarrow B(2) \]
\[\vdots \]
\[(10^7+2) \ \ B(10^7-1) \rightarrow B(10^7) \]
\[\vdots \ (10^7+3) \ \ B(10^7) \]

The second kind of boundary-affirming statement besides (3) is this:

\[(11) \ (\exists n)(B_n \& \neg B_{n+1}) \]

(There is an n such that an n-haired person is bald and it’s not the case that an (n+1)-haired person is bald.)

And this statement, obviously, is just as problematic as (3) with respect to the robustness of vagueness. Remember: robustness means that there is no fact of the matter about semantic transitions.

But the way to handle statements (10) and (11), within the general approach we are here following, is the same as with (2) and (3), by the same reductio line of reasoning. They and their strong negations are neither true nor false; thus the
weak negations of them and their strong negations are all true.

The sort of semantics that one would want, for a language employing vague predicates and governed by this sort of logic, would be one that not only assigns no truth value to certain statements—viz., statements like (1)-(5), (10), and (11)—but also assigns truth values to statements in a sorites sequence vaguely, rather than in any precise way. That, in turn, means that the semantical notions of truth and falsity will be robustly vague as well—and hence that the metalanguage in which the semantics is given will itself be subject to the same logic. And so forth, all the way up the metalinguistic hierarchy.

We can now address our four forms of the sorites argument, (Q), (Q2), (C), and (C2). Concerning (Q), the thing to say is that the key premise, viz.,

\[(2) \quad (n)(Bn \supset Bn+1),\]

is neither true nor false. Moreover, within the object language we have the expressive resources to say what one might initially have tried to say with (2), i.e., to deny the existence of any sharp semantic transitions:

\[(7) \quad \neg(\exists n)(Bn \& \neg Bn+1).\]

And (2) itself is to be denied, of course, but this way:

\[(6) \quad \neg(n)(Bn \supset Bn+1),\]

which does not entail either

\[(3) \quad (\exists n)(Bn \& \neg Bn+1)\]

or

\[(11) \quad (\exists n)(Bn \& \neg Bn+1).\]

Exactly analogous things are to be said about argument (Q2), vis-a-vis its key premise (10).

With respect to argument (C), the natural-looking approach is to advert to the fact that truth is itself robustly vague on this picture (mirroring the robust vagueness in the object language), and thus that the truth predicate in the metalanguage is subject to just the same logic operative in the object language. So consider the all the conditional premises in (C), and consider the metalinguistic statements

\[(12) \quad \text{Every premise of (C) is true}\]

and

\[(13) \quad \text{Some premise of (C) is not true}.\]

The logic of vagueness applies to these, in the metalanguage. So, by reductio reasoning we can argue that both (12) and (13) are both neither true nor false, and
hence that (C) is not sound. And the same goes for argument (C2), with respect to the analogs of (12) and (13):

(14) Every premise of (C2) is true
(15) Some premise of (C2) is not true.

This mode of response to arguments (C) and (C2), involving the metalinguistic statements (12)-(15), has an analog concerning the respective object-language statement obtained by conjoining all conditional premises of (C) and (C2) respectively:

(16) \([B(0) \supset B(1)] \& [B(1) \supset B(2)] \& \ldots \& [B(10^7-1) \supset B(10^7)]\]
(17) \([B(0) \rightarrow B(1)] \& [B(1) \rightarrow B(2)] \& \ldots \& [B(10^7-1) \rightarrow B(10^7)]\]

Statements (16) and (17) both are neither true nor false. Hence their strong negations too are neither true nor false. So their weak negations are true, and so are the weak negations of their strong negations.

So a logic with the features I have described evidently blocks the two quantificational versions of the sorites argument and also the two conditional versions, while also avoiding any commitment to sharp semantic boundaries. You might have a residual sense, though, that this way with the sorites has been rather too quick, and that there is still sorites-related trouble lurking. If so I think you are right, for reasons I will spell out in Section 4. First, though, let us ask about semantics for the approach to logic here proposed.


In logical theory, treatments of semantics typically focus primarily on the task of giving a recursive truth characterization for a formal language. One standard way to do this is model-theoretically: define truth in a model, and then define truth simpliciter as truth in the intended model. Truth in a model, in turn, is the central notion used to characterize key logical properties and relations for statements of the language: logical truth, logical consistency, and the logical-consequence relation.

The other standard approach, more directly in the spirit of Tarski, is to characterize truth homophonically — or at any rate, quasi-homophonically (as I'll put it). A literal homophonic truth characterization employs a metalanguage that is a direct extension of the object language, minimally enriched with key semantical vocabulary — notably a satisfaction predicate.\(^\text{13}\) The base clauses do not advert to the extensions of primitive terms (as in the model-theoretic approach). Rather, they use the terminology whose semantics they are specifying, as in this schema for monadic predicates:

For any object o, o satisfies ‘F’ iff Fo.\(^\text{14}\)
A quasi-homophonic truth characterization is much like a fully homophonic one, except that the operative metalanguage is a modestly richer extension of the object language. In the typical case, the metalanguage is a fragment of natural language, whereas the object language is a formal language; the metalanguage uses ordinary-language logical vocabulary: 'and', 'or', 'every', 'some', etc.

Prima facie, the model theoretic approach to semantics is inherently ill-suited for dealing with the logic of vagueness. For, models are certain kinds of sets; and sets, as noted already, are paragons of precision. The homophonic approach, on the other hand, does not face this problem. Moreover, prima facie one would expect it to do what needs doing, if the robustness of vagueness is to be accommodated: viz., to assign semantic values to sentences of the object language in a genuinely vague—i.e., robustly vague—way.

On the other hand, as Grandy (1986) points out, an adequate homophonic truth characterization can leave undetermined certain key logical properties and relations, even though those properties and relations are derivable, within the operative metalanguage, from the truth characterization. For, the derivation can depend crucially on the logic of the metalanguage itself, and the same truth characterization, with all the same "target biconditionals" of the form \([T(\phi) \leftrightarrow \psi]\) still derivable from it, might be formulable in a metalanguage governed by a different logic. Grandy writes:

> It has been claimed that...we learn about the logical forms of sentences by giving a [homophonic] truth theory for the language... . It certainly appears that...we learn from the truth theory more than the biconditionals—for example we can easily derive within standard truth theories that any specific instance of excluded middle is true; indeed we can derive that they are all true. Surely this information is about logical form.

> But how much of it comes from the truth theory? We can divide the derivation of the sentence \(T(\text{'A v -A'})\) into two parts, the first consisting of a derivation of a biconditional with that on the left and A v -A on the right, the second consisting of a derivation of A v -A followed by the result desired. The truth theory enters only into the first portion. To put the point more strongly, one can give a perfectly adequate truth definition of a classical language using an intuitionistic metalanguage, and in that case the result about excluded middle is clearly not forthcoming. (Grandy 1986, p. 181)

Given (i) Grandy's observation that a homophonic truth characterization can underdetermine the logical properties and relations of sentences of the language, and (ii) the observation in Section 2 that the metalanguage too will be subject to the non-classical logic of vagueness (since truth itself is robustly vague), one should approach the project of giving such a truth characterization with suitably modest expectations. The truth characterization is not likely to settle questions about logical properties or relations, independently of the logic operative in the metalanguage. The appropriately modest expectation, rather, is this: if the logic of the metalanguage has the characteristics described in Section 2, then the truth characterization will interact with this metalinguistic logic to yield the desired non-classical logical features for object-language sentences.
With this pared-down prior expectation in force, let me now set forth a quasi-homophonic truth characterization for a first-order language employing robustly vague predicates. Here is a very simple language of this kind:

Open formulas:

(1) Atomic open formulas: ‘(x is bald)’, ‘(x is a heap)’.

(2) Molecular open formulas:
   (a) If φ is an open formula, then so are ¬φ and φ.
   (b) If φ and ψ are open formulas, then so are (φ & ψ) and (φ v ψ).

Sentences (i.e., closed formulas):

(1) General sentences: If φ is an open formula, then (x)φ and (∃x)φ are sentences.

(2) Molecular sentences:
   (a) If φ is a sentence, then so are ¬φ and φ.
   (b) If φ and ψ are sentences, then so are (φ & ψ) and (φ v ψ).

Definitional abbreviations:

(φ ⊃ ψ) =df (¬φ v ψ);
(φ → ψ) =df (¬φ v ψ);
(φ = ψ) =df ((φ ⊃ ψ) & (ψ ⊃ φ));
(φ ↔ ψ) =df ((φ → ψ) & (ψ → φ)).

A corresponding quasi-homophonic truth characterization, which first simultaneously characterizes satisfaction and “dissatisfaction” and then simultaneously characterizes truth and falsity, goes as follows.15

Satisfaction and dissatisfaction:

For any object o and any atomic open formula φ,

(1a) o satisfies φ iff: either (i) φ = ‘(x is bald)’ and o is bald, or (ii) φ = ‘(x is a heap)’ and o is a heap;

(1b) o dissatisfies φ iff: either φ = ‘(x is bald)’ and o is not bald, or (ii) φ = ‘(x is a heap)’ and o is not a heap.

For any object o and any open formula φ,

(2a) o satisfies ¬φ iff o dissatisfies φ;

(2b) o dissatisfies ¬φ iff o satisfies φ;

(3a) o satisfies ¬φ iff it’s not the case that o satisfies φ;

(3b) o dissatisfies ¬φ iff o satisfies φ.

For any object o and any open formulas φ and ψ,

(4a) o satisfies (φ & ψ) iff: o satisfies φ and o satisfies ψ;

(4b) o dissatisfies (φ & ψ) iff: either o dissatisfies φ or o dissatisfies ψ;

(5a) o satisfies (φ v ψ) iff: either o satisfies φ or o satisfies ψ;

(5b) o dissatisfies (φ v ψ) iff: o dissatisfies both φ and ψ.
Truth and falsity:

If $\phi$ is an open formula, then

1a) $(x)\phi$ is true iff every object satisfies $\phi$;
1b) $(x)\phi$ is false iff some object dissatisfies $\phi$;
2a) $(\exists x)\phi$ is true iff some object satisfies $\phi$;
2b) $(\exists x)\phi$ is false iff every object dissatisfies $\phi$.

If $\phi$ is a sentence, then

3a) $\neg\phi$ is true iff $\phi$ is false;
3b) $\neg\phi$ is false iff $\phi$ is true;
4a) $\neg\phi$ is true iff it's not the case that $\phi$ is true;
4b) $\neg\phi$ is false iff $\phi$ is true.

If $\phi$ and $\psi$ are sentences, then

5a) $(\phi \& \psi)$ is true iff $\phi$ is true and $\psi$ is true;
5b) $(\phi \& \psi)$ is false iff either $\phi$ is false or $\psi$ is false;
6a) $(\phi \lor \psi)$ is true iff either $\phi$ is true or $\psi$ is true;
6b) $(\phi \lor \psi)$ is false iff both $\phi$ and $\psi$ are false.

Quasi-homophonic or purely homophonic truth characterizations for richer formal languages with vague terms would be constructed analogously, modifying the standard Tarskian approach along the same lines.

The appropriate condition of material adequacy we want, for such a truth characterization, is that for any sentence $\phi$ of the object language, three biconditionals of the following form are derivable, in the metalanguage, from the truth characterization. (Schema (N) is so labeled because it covers the case where the given sentence is neither true nor false.)

(T) $T(\langle\phi\rangle) \leftrightarrow \phi$
(F) $F(\langle\phi\rangle) \leftrightarrow \neg\phi$
(N) $[\neg T(\langle\phi\rangle) \& \neg F(\langle\phi\rangle)] \leftrightarrow (\neg\phi \& \neg \neg\phi)$.

Assuming that the metalanguage is itself governed by the logic of vagueness sketched in Section 2, a truth characterization along these lines should underwrite the appropriate semantic status for sorites-involving statements (2), (3), (10), and (11).

(2) $(n)(Bn \supset Bn+1)$,
(3) $(\exists n)(Bn \& \neg Bn+1)$,
(10) $(n)(Bn \rightarrow Bn+1)$,
(11) $(\exists n)(Bn \& \neg Bn+1)$.

Each of these, and their respective strong negations, should turn out neither true nor false; and the weak negations of these statements and their strong negations will turn out true. Statement (10), for instance, is true iff

for every number $n$, $n$ satisfies $(Bn \rightarrow Bn+1)$
iff

for every number n, either it’s not the case that n satisfies B, or n+1 satisfies B

iff

for every number n, either it’s not the case that a person with n hairs on his head is bald, or a person with n+1 hairs on his head is bald.

In our metalanguage, however, neither this latter statement nor its strong negation obtain. What holds in the metalanguage, rather, are its weak negation and also the weak negation of its strong negation:

(18) It’s not the case that for every number n, either it’s not the case that a person with n hairs on his head is bald, or a person with n+1 hairs on his head is bald.

(19) It’s not the case that not for every number n, either it’s not the case that a person with n hairs on his head is bald, or a person with n+1 hairs on his head is bald.

And so, by the conjunction of (18) and (19) plus the appropriate instance of schema (N), statement (10) turns out to be neither true nor false.

But although things apparently do work out as wanted regarding the semantic status of statements like (2), (3), (10), and (11), it is clear that—just as expected—the outcome is not a product of the truth characterization by itself, but rather a joint product of the truth characterization and the logic operative in the metalanguage. This point can be made vivid by considering how things turn out if we make different assumptions about the nature of the metalanguage.

Suppose, for instance, that the metalanguage is governed by classical logic. Then the two object-language negation symbols will, in effect, collapse into one another: they will be equivalent. (The connectives ‘→’ and ‘→’ will collapse into one another too, as will ‘≡’ and ‘≡’.) For any open formula φ and any object o, o will either satisfy or dissatisfy φ, and hence will either satisfy or dissatisfy φ. For any sentence φ, either φ or ¬φ will be true. Both sides of the ‘↔’ in schema (N) will be false for each sentence φ. As for sorites-related statements, either (2) or (3) will be true; likewise, either (10) or (11) will be true.

Or suppose instead that the metalanguage is governed by some wimpy nonclassical logic—say, the kind that results from standard supervaluationism model-theoretic semantics. Then although some predicates, including perhaps our predicate ‘B’, might be such that certain objects neither satisfy nor dissatisfy them, still there will be two precise boundaries, in sorites sequences of this kind: a boundary between the true statements and the ones lacking truth value, and another boundary between the latter and the false statements. Thus, if the baldness sequence has members that are neither true nor false, then although statements (2) and (3) will each turn out neither true nor false, (10) will turn out
false and (11) will turn out true.

So although our homophonic truth characterization evidently works properly under suitable assumptions about the logic of the metalanguage, it works differently under different such assumptions. Given this outcome, and given the unpromisingness of model-theoretic semantics, it appears that one important implication of the robustness of vagueness is the need for some new kind of semantical theory, different from the model-theoretic kind and broader than a homophonic truth characterization. An especially salient task for such a semantics would be to give some suitable, non-model-theoretic, account of key logical properties and relations: logical truth, logical consistency, and the logical consequence relation. What such a semantical theory might look like is a question I will leave largely open, although my subsequent remarks (especially in Section 6) will have some bearing on the matter.

Of course, since there is currently no such semantical theory to underwrite the logic of robust vagueness, and since it is quite unclear even what sort of theory to look for, the worry also arises that perhaps no such theory is possible—that perhaps genuine, thoroughly robust, vagueness is ultimately incoherent, and hence impossible. And in fact the problem of incoherence gets dramatically forced upon us by a version of the sorites paradox against which the nonclassical logic I have presented seems impotent, a version which seems to establish both the impossibility of vagueness and the underlying incoherence of this logic itself. I turn next to that.

4. Thesis: Vagueness is Impossible.

I now commence the three-part dialectical investigation I described in the introduction. In this section I take on the role of advocate for the Thesis—the contention that vagueness is impossible. I base this contention on a form of the sorites paradox I will call the forced-march sorites—so named because it is designed to force us, one step at a time, into a separate verdict on each successive pair of adjacent items in a sorites sequence. The reasoning proceeds as a linked sequence of subarguments. For the sorites sequence of baldness statements B(0), B(1), ..., B(107), the argument goes as follows.

(A1) Consider the true statement B(0), together with its right neighbor B(1). What are the possibilities concerning the semantic status of B(1)? Allow as many different possible kinds of semantic status for B(1) as you like—e.g., (1) true; (2) false; (3) neither true nor false; (4) neither true, nor false, nor neither true nor false; (5) indefinite whether true or false; (6) indefinite whether true, false, or indefinite whether true or false; ...; etc. No matter now many such possibilities there might be (even infinitely many), either B(1) has the same semantic status as B(0) itself—viz., truth—or
else B(1) differs from B(2) in semantic status. But if B(0) and B(1) differ in semantic status, then there is a sharp semantic boundary between them—which is incompatible with the robustness of genuine vagueness. Hence B(1) is true.

(A2) Consider the true statement B(1), together with its right neighbor B(2). [Etc. for B1) and B(2), as per subargument (A0).] Hence B(2) is true.

. . .

(A107) Consider the true statement B(107-1), together with its right neighbor B(107). [Etc. for B(107-1) and B(107), as per the preceding subargument.] Hence B(107) is true.

This argument is very difficult to fault. Each subargument is an instance of the same valid argument-form. And, within each successive subargument (Ai), the premise

Either B(i) and B(i+1) have the same semantic status, or else they differ in semantic status

is surely true, as is the premise that a difference in semantic status between B(i) and B(i+1) would be incompatible with the robustness of vagueness. Thus, for each subargument (Ai), if B(i) is true then (Ai) is not only valid but also sound. So, since subargument (A1) is sound, so is (A2); since (A2) is sound, so is (A3); etc. Thus all the subarguments are sound, and hence the argument as a whole is sound.

Given the soundness of forced-march sorites arguments, the argument for the impossibility of vagueness goes as follows. If vagueness is possible, then it is possible for there to be a sorites sequence with true statements at the beginning, false statements at the end, and no sharp semantic transitions anywhere in the sequence. But for any sorites sequence that commences with true statements, there is a sound forced-march sorites argument showing that every statement in the sequence is true. So if vagueness is possible, then there are sorites sequences containing statements that are both true and false, which is impossible. Hence, vagueness itself is impossible.

Now, the valiant defender vagueness (who I’ll henceforth call Val) can be expected to reply to this argument by invoking the official response dictated by the logic of vagueness I described in section 2. That is, Val can focus on the conjunction of the critical premises in the respective sub-arguments, and on the corresponding universally quantified statement:

(20) Either B(0) and B(1) have the same semantic status or they differ in semantic status; and either B(1) and B(2) have the same semantic
status or they differ in semantic status; and ...; and either B(10^7)-1 and B(10^7) have the same semantic status or they differ in semantic status.

(21) For any n, either B(n) and B(n+1) have the same semantic status or they differ in semantic status.

Val can say that the weak negations of (20) and (21) are true; that the weak negations of their strong negations are also true; and thus that (20) and (21) themselves are neither true nor false. Similarly, concerning the issue of the soundness of the various subarguments, Val can focus on the conjunction of the respective soundness claims, and on the corresponding universally quantified statement:

(22) Subargument (A0) is sound; and subargument (A1) is sound; and ...

; and subargument (A10^7) is sound.

(23) For any n, argument (An) is sound.

Again, Val can say that the weak negations of (22) and (23) are true; that the weak negations of their strong negations are also true; and thus that (22) and (23) themselves are neither true nor false.

But this move, which ducks the forced march by refusing to say anything specific about the respective subarguments considered individually, can be seen to be incoherent once we hold Val’s feet to the fire, in the following way. Take all the pairs of adjacent statements <Bn, Bn+1> from the sorites sequence. Select any single pair Bi and Bi+1, and pose this question: “Do Bi and Bi+1 have the same semantic status?” What can Val say, when forced to confront this question? Not that it is meaningless; for, the nonstandard logic Val advocates provides no conceptual room for that. Not that it lacks an answer; for, Val’s non-standard logic provides no conceptual room for that either (even though it does allow for semantic statuses other than truth and falsity). Not that it lacks a yes-no answer, and instead has some other kind, e.g.,

It’s not the case that Bi and Bi+1 have the same semantic status, and it’s not the case that Bi and Bi+1 do not have the same semantic status.

For, to affirm this would be to posit a (higher-order) semantic boundary between Bi and Bi+1, thus flouting the robustness of vagueness.17 In short, when pressed concerning Bi and Bi+1, Val has no option but to affirm this disjunction:

Either Bi and Bi+1 have the same semantic status, or else they differ in semantic status.

Now continue selecting, one at a time (and in any order), all the other pairs of statements <Bi, Bi+1>. For each such pair, the argument just given again applies. The upshot is that Val, when pressed about each pair individually, one at a time, will eventually affirm every instance of the following open sentence,
for $0 \leq n \leq 10^7$:

(24) Either $B_n$ and $B_{n+1}$ have the same semantic status, or else they differ in semantic status.

But of course, it is logically incoherent to affirm every instance (24), on the one hand, but on the other hand to deny (either strongly or weakly) both their conjunction (20) and the corresponding universal quantification (21). So, given that Val really has no choice but to affirm every instance of (24), Val’s affirmation of (20)-(23) is logically incoherent.

So, not only does the forced-march sorites argument withstand Val’s attempt to block it, but forcing Val herself to separately confront each pair $<B_i, i+1>$ reveals the underlying logical incoherence of her own position. Moreover, that incoherence also undermines the earlier responses to quantificational and conditional sorites arguments, as set forth in Section 2. Take argument (C), for instance. If we force Val to separately confront each individual pair of conditionals $<(B_i \supset B_{i+1}), (B_{i+1} \supset B_{i+2})>$, she will have no choice but to admit that either the two conditionals have the same semantic status, or else they differ in semantic status. In addition, she will have no choice but to admit that they have the same semantic status, since a difference would reflect a semantic boundary either between $B_i$ and $B_{i+1}$ or between $B_{i+1}$ and $B_{i+2}$—which is contrary to the robustness of vagueness. So, since the initial conditional premises in argument (C) are true, Val has no choice, when pressed, but to affirm of each individual conditional premise that it is true.18 Hence, in affirming that their conjunction,

(16) $[[B(0) \supset B(1)] \& [B(1) \supset B(2)] \& ... \& [B(10^7-1) \supset B(10^7)]]$,

and the corresponding universal quantification,

(2) $(n)(B_n \supset B_{n+1})$,

are both neither true nor false, Val adopts a logically incoherent position.

Given that genuine vagueness, if there could be such a thing, would be robust, forced-march sorites reasoning establishes that genuine vagueness is impossible. And the sort of non-standard logic that seems required to block such reasoning turns out, on reflection, to be incoherent.

5. Antithesis: Vagueness is Actual (and Hence Possible).

If indeed vagueness is impossible, then of course it is nonexistent: there are no vague objects or vague properties in the world; no vague concepts employed in human thought; and no vague terms in human language. This means, in turn, that wherever we ordinarily think there is vagueness, there must actually be certain hidden facts, to which we have little or no epistemic access, in virtue of
which there is really complete, utter, precision.\textsuperscript{19}

In this Section I will argue that there are no such hidden facts, and that vagueness is therefore actual (hence possible). I will maintain not that the putative existence of such facts is incoherent, but rather that the empirical evidence against them is enormously strong. The empirical case involves three interrelated, mutually supporting, considerations.

First is the content of our semantic intuitions, as competent language users, about matters of vagueness and precision. For instance, take the term ‘tall’, as applied relative to some specific reference class of humans (say, Caucasian males). When we consult semantic intuition, it seems obvious there is no single precise minimum value n such that a Caucasian male who is n millimeters in height is tall. It seems obvious that nothing about the world in itself, or about our concept of tallness, or about the semantics of the term ‘tall’, uniquely sanctions any specific value n, over against numerous others—and hence that any such choice would be utterly arbitrary.

These semantic intuitions, especially since they are so persistent and so universal, provide strong empirical evidence that there is no precise, non-arbitrary, minimum height for tallness. Semantic intuitions of competent speakers constitute an important form of empirical data, \textit{vis-a-vis} hypotheses and theories about the workings of human concepts and the semantics of our terms. (The evidential role of such intuitions is quite analogous to the role of competent speakers’ intuitions about grammaticality and syntactic ambiguity, \textit{vis-a-vis} empirical hypotheses and theories about natural-language syntax.) The empirical evidence they provide is of course defeasible: semantic intuitions, like other intuitions, can be mistaken. Nevertheless, there is an epistemic presumption in their favor, particularly when the intuitions are both persistent and universal; for, under those conditions especially, they probably emanate from speakers’ semantic competence as language users and their cognitive competence as concept users, and hence are probably correct.

The second consideration telling against precise boundaries for seemingly vague notions like tallness is that we currently cannot even begin to conceive or imagine—not even in a very sketchy way—what \textit{kinds} of putative hidden facts could do the job required of them: \textit{viz.}, to combine with more familiar facts to generate a precise, non-arbitrary, boundary. That we cannot even conceive what such hidden facts could be like is itself substantial evidence that they don’t exist. Once again, it is indeed \textit{empirical} evidence, and hence defeasible; for, what we cannot now conceive might be actual, and hence metaphysically possible, anyway. (Before Einstein, who could conceive of the relativity of simultaneity, or of curved spacetime?) But unless and until some positive account emerges, our present inability to form any positive conception of hidden precisification-facts strongly warrants concluding that there aren’t any.\textsuperscript{20}

Third, when one looks to contemporary science—physics and/or any of the special sciences—one evidently finds nothing there that lends any theoretical
support to the hypothesis that seemingly-vague notions like tallness really have perfectly precise boundaries. Quite the contrary: given the broad outlines of contemporary physics, it would appear that even a complete physics-level characterization of the world would fail to provide any non-arbitrary way to precisely delimit the extension of terms like 'tall'; and nothing one can point to in the special sciences suggests that facts specifiable in special-science vocabulary could play this role either.\textsuperscript{21} As far as one can tell, our best empirical theories of the world just do not posit hidden facts that determine precise boundaries where there is apparent vagueness.

The considerations just mentioned, all epistemically potent in their own right, strongly reinforce each other as well. Together they add up to a tremendously strong empirical case against the existence of hidden, boundary-precisifying, facts—and thereby in favor of the reality of vagueness.

In response to this argument, the stubborn denier of vagueness might try a fallback position. On the one hand he continues to insist that there is no genuine vagueness: real, robust, vagueness is impossible. On the other hand he admits that the argument against hidden boundary-precisifying facts has some force and so he opts for some wimpier logico-semantic treatment—the degrees-of-truth approach or supervaluationism—of notions we ordinarily regard as vague. Unlike the original propounders of these treatments, he would not be claiming to give an account of vagueness itself. Rather, he would be claiming that the terms and concepts we normally consider vague are not really vague at all (since genuine, robust, vagueness is impossible), but are actually only wimpily pseudo-vague. This proposal, he maintains, preserves logical coherence for these terms and concepts; and positing wimpier pseudo-vagueness allows us to avoid any commitment to a sharp line of demarcation between, e.g., those persons who are bald and those who are not bald, or between those Caucasian males who are tall and those who are not tall, etc.

But a moment's reflection reveals that this fallback reply is not tenable. The problem is that even wimpier pseudo-vagueness would require the kinds of hidden precisification-facts I have just argued against. Relative to the baldness sequence, for instance, certain hidden facts would have to combine with ordinary facts to determine a unique natural number n such that Bn is true but Bn+1 is not true; and a unique natural number m such that Bm is not false but Bm+1 is false. The argument against hidden line-drawing facts extends to this kind too, no less than to the kind that would preserve classical logic.\textsuperscript{22}

Given (i) our semantic intuitions about matters of vagueness, (ii) our present inability to conceive what hidden precisification-facts could be like, and (iii) the fact that current science evidently does not posit any such facts or enlighten us about how there could be any, the empirical case against the existence of these putative facts is overwhelming. This negative case also extends to any fallback anti-vagueness position invoking degrees of truth, supervaluationism, or other comparably wimpier logico-semantic approaches.

Initially it seems that the Thesis and the Antithesis are exclusive and exhaustive, so that one or the other must be right. If so, then perhaps we finally have no choice but to acknowledge that the aprioristic argument for the Thesis trumps the empirical argument for the Antithesis, and to insist that there simply must be hidden precisification facts wherever we ordinarily think there is vagueness. Given the strength of each argument, however, we have reason to seek out a position that largely accommodates them both, and that somehow avoids fully accepting or fully rejecting either the Thesis or the Antithesis. In this section I will propose such a position.

When caught philosophically between a rock and a hard place, the appropriate initial strategy is to make a distinction. In this case, the distinction we want is between two potential kinds of vagueness. On the one hand is vagueness in certain objects, properties, or other entities in the mind-independent, discourse-independent, world. (Henceforth, in order to emphasize that I mean the world in itself, independently of how we happen to think about it or talk about it, I will employ Hilary Putnam’s capitalization convention. On the one hand, then, is vagueness in THE WORLD, and in certain OBJECTS, and/or PROPERTIES, and/or other ENTITIES.) On the other hand is vagueness in human thought, and/or in human language.

A preliminary formulation of the Synthesis I want to propose, over against both the Thesis and the Antithesis, is this:

Vagueness in THE WORLD is impossible, but vagueness in thought and in language is actual (and hence possible).

But there needs to be more to the Synthesis than this, because the argument in Section 4 was that vagueness is impossible simpliciter—not just that it is impossible in THE WORLD. As we saw, the attempt to block that argument by resorting to the non-classical logic of Section 2 is ultimately logically incoherent. THE WORLD, of course, cannot be logically incoherent (although it could certainly turn out to be incomprehensible). Thought and language, on the other hand, can be logically incoherent, without thereby being meaningless. So the Synthesis should assert that vagueness in thought and language is both actual and incoherent. Thus the official Synthesis is this:

Vagueness in THE WORLD is impossible; vagueness in thought and in language is incoherent, and yet is actual (and hence possible) anyway.

I will devote the remainder of this section to an exposition and defense of the Synthesis position regarding vagueness in thought and in language. (I will briefly return to vagueness and THE WORLD in Section 7.)

Even though vagueness is logically incoherent, it does not follow that vague concepts or vague terms are impossible. Rather, as long as the
incoherence somehow remains well insulated, instead of propagating itself destructively through our thought and discourse, there is no reason why it cannot be present there—dormant, so to speak. (For purposes of this paper I can leave it open how best to cash the general notion of logical incoherence, and also the more specific notion of insulated logical incoherence. Roughly, a concept is logically incoherent if someone who employs it correctly thereby becomes committed, at least implicitly, to accepting statements that jointly entail a contradiction. The incoherence is insulated if there are features of thought and language that systematically prevent the commitment from surfacing explicitly.)

Logical incoherence is usually quite a bad thing, of course, because its effects are typically so virulent. But one should not infer that it is always harmful or debilitating. In fact, here is an argument to the contrary, with respect to the kind of incoherence manifested by vagueness (henceforth, v-incoherence). Vagueness is actual, and is often a highly useful and desirable attribute of human concepts and terms. To a large extent, its utility stems from its robustness. But robustness is also the source of v-incoherence. Therefore, v-incoherence is not a bad thing; although most kinds of logical incoherence are fatally malignant in their effects on thought and discourse, v-incoherence is benign.

What logico-conceptual mechanisms operate to insulate v-incoherence, keeping it dormant and preventing it from generating destructive effects? Part of the answer, I take it, is that human categorization schemes evolve pragmatically, in such a way that frequently-encountered objects, events, and situations tend to wind up partitioned rather cleanly within our operative categories, rather than winding up within the penumbral periphery where vagueness starts to matter. As long as the partitioning goes cleanly, vagueness does not intrude and we can simply rely on classical logic.

But the insulatory mechanisms need to be more resilient than this, because penumbral cases do arise occasionally, both in ordinary life and in theoretical inquiry. (They arise all the time in philosophical inquiry.) In such cases, it seems plausible to suppose, we employ (often tacitly) the sort of logic I sketched in Section 2. Two complementary factors are involved in this. First we reject, via weak negation, statements like the following, associated with the three kinds of sorites arguments considered earlier (viz., quantificational, conditional, and forced-march versions):

(2) \( (n)(Bn \Rightarrow Bn+1) \)
(10) \( (n)(Bn \rightarrow Bn+1) \)
(16) \( [[B(0) \Rightarrow B(1)] \& [B(1) \Rightarrow B(2)] \& \ldots \& [B(10^7-1) \Rightarrow B(10^7)]] \)
(17) \( [[B(0) \rightarrow B(1)] \& [B(1) \rightarrow B(2)] \& \ldots \& [B(10^7-1) \rightarrow B(10^7)]] \)
(20) Either B(0) and B(1) have the same semantic status or they differ in semantic status; and either B(1) and B(2) have the same semantic status or they differ in semantic status; and \ldots; and either B(10^7-1)
and B(10^7) have the same semantic status or they differ in semantic status.

(21) For any n, either B(n) and B(n+1) have the same semantic status or they differ in semantic status.

We also reject, via weak negation, the strong negations of such statements, since these are logically equivalent to statements asserting the existence of sharp boundaries between vague categories. Second, we steadfastly refuse to take a stand on each separate pair of adjacent items in a sorites sequence; i.e., we refuse to be subjected to forced-march querying about these individual pairs. If pressed, we merely say "There's no fact of the matter about category-transitions," and we refuse to be pressed further.

Now, as I argued in Section 4, this refusal reveals that the logic described in Section 2 is itself incoherent, at bottom. For, we can provide no cogent rationale for denying (either strongly or weakly), of any particular pair of adjacent items in a sorites sequence, that a query about the comparative status of those two items has a correct answer; yet once we acknowledge this, the nascent incoherence of our non-standard logic quickly comes to the surface. But if the real point of the logic of vagueness is not to eschew incoherence but merely to insulate it, then this nascent incoherence will not be a problem—provided that it remains nascent. And it is easy enough to keep it that way: one just stubbornly maintains one's refusal to answer those persistent queries about adjacent pairs in the sorites sequence. One asserts oneself, and refuses to be intellectually cajoled into the forced march. Praxis trumps theoria.

As we saw in Section 3, a Tarski-style truth characterization can be given for an object language governed by the logic described in Section 2; and if the metalanguage is governed by that logic too, then object-language statements like (2), (10), (16), (17), (20), and (21) all turn out neither true nor false, as desired. These results about semantics further underscore the fact that the non-standard logic of vagueness can be employed in a way that effectively insulates its own underlying incoherence, thereby insulating v-incoherence as well.

To summarize: The argument in Section 4 shows that vagueness in THE WORLD is impossible. This argument cannot be deflected by appeal to the non-classical logic of Section 2, because (as was also shown in Section 4) vagueness and its logic are, at bottom, incoherent. With respect to thought and language, however, incoherence does not entail impossibility. So, given (i) the argument in Section 5 that vagueness is actual, and (ii) the fact that v-incoherence can be insulated so effectively as to be benign rather than malignant, vagueness in thought and language is both incoherent and actual.

7. Some Implications.

I will conclude by briefly discussing several philosophical consequences of
the proposed Synthesis. The first involves what might be called *logical semantics*, i.e., those aspects of semantic theory that provide the semantic underpinnings for logic. Given the Synthesis, an adequate logical semantics would probably turn out to be a two-tiered affair. At the first tier would be a logic that ignores matters of vagueness (perhaps just classical logic), with an accompanying semantical characterization of key notions like logical truth, logical consistency, and the logical consequence relation. A model-theoretic approach would be appropriate here. At the second tier would be an expanded logic along the lines of Section 2, containing weak negation in addition to strong negation, plus the conditional and biconditional connectives ‘→’ and ‘↔’ definable via weak negation and disjunction. The accompanying semantics, at this second tier, would presumably acknowledge explicitly that the expanded logic is semantically incoherent by first-tier standards; and would also point out why, and how, this incoherence gets insulated rather than having malignant effects. In addition, it might also provide expanded characterizations of logical truth, logical consistency, and logical consequence. A model-theoretic approach would *not* be appropriate here, presumably.25 What such expanded characterizations of these notions might look like, then, is an important residual question for logical semantics.

Second is the import of the Synthesis for ontology. If indeed there are no vague OBJECTS, PROPERTIES, or other ENTITIES in THE WORLD, then a correct catalog of what there IS would exclude many or most of the entities whose existence seems presupposed in human discourse. Not only are there no such PROPERTIES as TALLNESS or BALDNESS, but there are no such ENTITIES as MOUNTAINS, DESKS, or even PEOPLE.26 For, if there were such entities, then there would be no fact of the matter about their precise spatio-temporal boundaries.27 But by the argument in Section 4, the notion of an OBJECT such that there is no fact of the matter about its boundaries, is at bottom incoherent. Any ONTOLOGY compatible with the Synthesis position will thus be quite radically at odds with the apparent ontological commitments of human discourse, including those of much scientific discourse.28

Third is another point about semantics, in light of this conclusion about metaphysics. A credible overall semantical theory, I take it, ought to allow for the genuine *truth* of statements of the kind we ordinarily regard as obviously and nonproblematically true. So, since such statements quite typically talk about vague objects like mountains, desks, and people, and since they quite typically predicate vague properties like tallness and baldness, a credible overall semantical theory should not construe the notion of truth as involving simple, direct, language/world connections between (i) the referential and predicative apparatus of our discourse, and (ii) OBJECTS and PROPERTIES. Rather, it should instead treat truth as involving a mode (perhaps various modes, in various contexts) of "correspondence" between language and THE WORLD that is considerably more subtle, and considerably less direct.29
Finally, and related to the last two points, is an observation about the metaphysics of semantics. As I argued in Section 2, where vagueness is involved in object-level discourse, truth itself is vague too; its vagueness mirrors the vagueness in the object language. So, since there are no vague PROPERTIES according to the Synthesis, there is no such PROPERTY as TRUTH either.30

These implications are among the reasons why I said in the introduction that the sorites paradox is much more fraught with import for metaphysics, semantics, and logic than is generally appreciated. With respect to the semantics and logic of vagueness, none of the standard approaches to the sorites work or are even in the ballpark of working; we evidently need a new kind of logical semantics, and at present nobody really knows what it should be like. And with respect to metaphysics, the sorites has very radical, yet seemingly unavoidable, ontological consequences. So let me end by reiterating, specifically with respect to the sorites paradox, the words of Quine: “Of all the ways of paradoxes, perhaps the quaintest is their capacity on occasion to turn out to be so very much less frivolous than they look.”31,32

Notes

1. See Kneale and Kneale (1962), who write:

From the explanations given by various writers of later antiquity it appears that some of the seven paradoxes specifically attributed to Eubulides were merely variants of others and that the list can probably be reduced to the four following items:

(1) The Liar. ‘A man says that he is lying. Is what he says true or false?’
(2) The Hooded Man, the Unnoticed Man, or the Electra. ‘You say you know your brother. But that man who came in just now with head covered is your brother, and you did not know him.’
(3) The Bald Man, or the Heap. ‘Would you say that a man was bald if he had only one hair? Yes. Would you..., etc. Then where to you draw the line?’
(4) The Horned Man. ‘What have you not lost you still have. But you have not lost horns. So you still have horns.’ (p. 114).

Item (3) is the sorites paradox. The other traditional example of it involves a heap of sand from which grains are removed one at a time. The Greek word ‘soreites’ means ‘one who heaps or piles up’.

2. By contrast, the other two kinds of paradox the ancients attributed to Eubulides—item (2) in note 1, involving intensional contexts, and item (4), involving the problem of nonbeing—have both received substantial philosophical attention.

3. Example: If, when you read or hear the word ‘theory’, you think ‘axiomatic formal system’, then you probably remain more within the grip of the positivist legacy, and thus of the Euclidean model, than you may realize. When I say that the positivist legacy perpetuates the idea that Euclidean mathematics is the model for human knowledge in general, I do not mean to deny the importance of the distinction between a priori knowledge and empirical knowledge. Rather, the point is that for positivists the paradigms of knowledge, on both sides of this epistemic divide, were axiomatic formal systems: axiomatized logical or mathematical theories on the one hand, and axiomatized physical theories on the
other hand.

4. This section is largely adapted from the section 'Robust vs. Wimpy Vagueness' in my (1990). Very similar objections to standard treatments of vagueness are raised by Sainsbury (1991a, 1991b) and Tye (this volume).

5. For instance Gouguen (1968-9); Fine (1975); Sanford (1975, 1976).

6. Here and throughout, I will be casual about use/mention niceties.

7. I take it that this is essentially what Sainsbury (1991a, 1991b) means by 'boundarylessness'. He means lack of any precise boundaries between different semantic statuses, for statements in the sequence.

8. One way to put this point is to say that the metalinguistic predicate 'is an eligible candidate-extension of' is itself vague. Could a supervaluationist acknowledge this fact, and somehow build it into his formal semantics for vagueness? It would appear not. For, supervaluationist semantics posits a set of eligible candidate-extensions. Sets are paragons of precision. So if the predicate 'is an eligible candidate-extension of' is itself vague, then there is no such set.

9. My proposed treatment of the logic of vagueness is similar to that of Michael Tye (1990, this volume), although our respective approaches to logical semantics are quite different.

10. Subsequent cross-references to displayed sentences will cite the numbers of these sentences as displayed directly, rather than as the sentences within displayed arguments. For instance, '(n)(Bn ⊃ Bn+1)' will be cited as statement (2), even though it occurs in argument (Q) above as premise (1).

11. Putnam (1983b) proposes using intuitionist logic to block sorites reasoning. In the logic I am proposing here, weak negation plays a role with respect to statements (2) and (3) somewhat similar to the role played, under Putnam's proposal, by intuitionist negation and intuitionist double negation. For a critique of Putnam see Read and Wright (1985); he replies in Putnam (1985). His position is usefully elaborated, in light of this exchange with Read and Wright, in Schwartz (1987). For a more recent critique see Schwartz and Troop (1991); he replies in Putnam (1991 pp. 413-14).

12. There might also turn out to be certain statements in the baldness sequence that lack truth value, under this vague truth-value assignment. But there needn't be. The crucial thing is that there be a robustly vague transition, from one semantic status to another, as one progresses along the baldness sequence. It matters little whether three semantic statuses are involved for the individual statements in the baldness sequence—true, false, and neither, with two robustly vague transitions—or whether, instead, there is only one robustly vague transition, directly from truth to falsity, without there being any statements in the baldness sequence that get assigned the semantic status "neither true nor false."

13. I use the phrase 'truth characterization', rather than the more common 'truth theory', because I doubt whether a homophonic or quasi-homophonic truth characterization necessarily deserves the honorific label 'theory'. Some grounds for doubting this will emerge in the course of this section.

14. I am here using the term 'homophonic', as is commonly done, for truth characterizations of the kind that Grandy (1986) calls "homophonic and homomorphic." Heteromorphic ones he describes this way:

   [I]n heteromorphic truth theories the goal is not to preserve the superficial form of the L sentence but to reveal its true underlying logical form.... In its usual setting, Russell's analysis of definite descriptions provides a means to define truth for a first-order language in a metalanguage without definite descriptions. (p. 180)

Homomorphic truth characterizations, by contrast, "smoothly pass the syntactic and semantic structure across the biconditional with minimal alteration" (p. 181).

15. The phrase 'iff', being short for 'if and only if' is to be understood in accordance with my earlier stipulation concerning the English expressions 'φ only if ψ' and
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'φ if ψ'. That is, it is short for (φ → ψ) & (ψ → φ), which is equivalent to (φ ↔ ψ). We want a metalinguistic conditional to be vacuously true when it's not the case that its antecedent is true.

16. Timothy Williamson has posed a problem I would like to set out and address. He observes that clause (4a) of my proposed truth/falsity characterization can be formalized this way:

\[(*)1\] \(T(¬φ') ↔ ¬T(φ')\).

Also, from schema (T) we have

\[(*)2\] \(T(¬φ') ↔ φ\).

From \((*)1\) and \((*)2\), by the kind of reasoning needed to derive (T), (F) and (N) from the Tarskian truth characterization, we should obtain

\[(*)3\] \(¬T(φ') ↔ ¬φ\).

One would also expect this to obtain, under the truth characterization:

\[(*)4\] \(¬T(φ') ↔ φ\).

But \((*)3\) and \((*)4\) yield, by the same sort of reasoning as before,

\[(*)5\] \(¬T(φ') ↔ ¬T(φ')\).

The problem Williamson poses is this: \((*)5\) seems to deny metalinguistic vagueness.

But although I am indeed committed to schemas \((*)1\)-(*)5), the fact that all instances of \((*)5\) obtain does not mean that the negation operators '¬' and '¬', behave the same way in general with respect to the truth predicate, and hence does not mean that metalinguistic vagueness is being repudiated. Consider, for instance, the following metalinguistic statement (with the variable 's' ranging over object-language statements in some sorites sequence like the baldness sequence):

\[(*)6\] \(s[T(s)] → T(s+1)]\).

Weak and strong negation work differently vis-a-vis \((*)6\), under my approach, because \((*)7\) holds but \((*)8\) does not, and likewise \((*)9\) holds but \((*)10\) does not:

\[(*)7\] \(¬(s)[T(s)] → T(s+1)]\)
\[(*)8\] \(¬(s)[T(s)] → T(s+1)]\)
\[(*)9\] \(¬(s)[T(s)] → T(s+1)]\)
\[(*)10\] \(¬(s)[T(s)] → T(s+1)]\).

And since \((*)7\) and \((*)9\) obtain, the truth predicate is indeed vague.

17. That this move would amount to positing a sharp boundary, of the kind incompatible with the robustness of vagueness, becomes quite clear when one considers the full set of pairwise questions. For some such questions, involving pairs near the beginning of the baldness sequence, the answer is a flat 'yes': both statements in the pair are true. So, if there is a pair <Bi,Bi+1> in the sequence such that Bi is true but the answer to the question about <Bi,Bi+1> is "It's not the case that they have the same semantic status, and it's not the case that they do not have the same semantic status," then we get a sharp semantic boundary between Bi and Bi+1.

18. John Tienson has asked whether the forced-march sorites paradox isn't just the conditional sorites "with your nose rubbed in it (i.e., taken seriously)." I would say yes.


20. It also constitutes reasonably strong empirical grounds for concluding that such facts are not even metaphysically possible.
21. Even if science posits no such precising facts in the mind-independent world, does cognitive psychology perhaps posit such facts "in the head," i.e., in language and thought? On the contrary, the psychological literature on human concepts and categories, influenced heavily by seminal work on prototype phenomena by Eleanor Rosch (e.g., 1973, 1975, 1978), evidently supports the contention that many concepts and categories are indeed vague. For further discussion of this literature and its bearing on vagueness, see Horgan (1990).

22. The stubborn denier of vagueness might try retreating still further, to this fallback position: (i) admitting that precisification is arbitrary; and (ii) claiming that the terms and concepts we ordinarily consider vague are really precise, albeit by virtue of arbitrary fiat. But except in special circumstances where arbitrary cutoff points get explicitly decided or legislated, people seem to employ vague terms and concepts without precisifying them, and without any need to do so. Thus, this last-ditch fallback position really requires positing a new brand of putative "hidden" facts—viz., ones in virtue of which certain arbitrary cutoff points get implicitly decided, without people's realizing it. All the earlier arguments against hidden precisifying facts now apply all over again, mutatis mutandis. Out of the frying pan, into the fire.

23. Examples worth mentioning in this connection, include (i) global logical incoherence in large databases, and (ii) global logical incoherence in science, like the incompatibility of quantum mechanics and general relativity theory. Also perhaps relevant is the lottery paradox: I believe that some ticket will win, while also believing, of each individual ticket, that it won't win.

24. In general the partitioning goes more cleanly for kinds, like Caucasian male and basketball player, than for properties like tallness. In practice, however, property attributions often get implicitly relativized to contextually relevant kinds, which facilitates partitioning.

25. Not, at any rate, if models are sets as traditionally understood in set theory. However, Tye (1990, this volume) proposes to alter traditional model theory by assigning vague sets to predicates as their extensions and counter-extensions. (These sets are supposed to genuinely, robustly, vague—not wimpily pseudo-vague like those of so-called "fuzzy set theory.") From the vantage point of the present paper, one serious drawback of Tye's approach is that it obscures the underlying logical incoherence of vagueness rather than acknowledging it; this in turn, generates the spurious appearance that there is no logical obstacle to vagueness in THE WORLD.


27. W. V. O. Quine describes well the vagueness of desks:

Who can aspire to a precise intermolecular demarcation of a desk? Countless minutely divergent aggregates of molecules have equal claims to being my desk... Vagueness of boundaries has sparked philosophical discussion in the case of desks because of their false air of precision. Mountains meanwhile are taken in stride. At bottom the two cases really are alike; our terms delimit the object to the degree relevant to our concerns.... [The] cases differ only in degree. (Quine 1985, pp. 167-168)

Similar remarks apply to persons, of course.

28. I myself favor an ontological position I call Parmenidean materialism; cf. Horgan (1991). A number of prominent neo-pragmatist, or “irrealist,” philosophers represent themselves not as advocating a radical ontology, but rather as repudiating traditional ontology altogether; see Dummett (1975, 1978), Goodman (1978), Putnam (1981, 1983a), and Rorty (1979, 1982). But irrealism, it seems to me, is actually an (extraordinarily radical) ontological position, the claims of its advocates to the contrary notwithstanding. Irrealism asserts that there is no mind-independent, discourse-independent, world (i.e., no WORLD) at all—there aren’t even any MINDS, either human or divine. It is thus
an ontology of NOTHINGNESS.

29. I spell out and defend one such approach to truth and ontology in several interrelated papers; cf. Horgan (1986a, 1986b, 1990, 1991). The central idea, motivated in part by considerations other than vagueness, is that truth is a normative attribute, viz., correct assertibility.

30. If truth is a normative attribute, as I have elsewhere maintained (cf. note 29), then this conclusion can be motivated on other grounds as well—the same sorts of grounds that motivate metaphysical irrealism in meta-ethics, vis a vis morally normative language. See Horgan and Timmons (forthcoming).

31. Quine (1966), p. 20. Ironically, this essay on the importance of paradoxes upheld philosophical tradition by completely ignoring the sorites paradox.

32. For helpful discussion, correspondence, and/or comments I thank John Ellis, Mark Heller, Mark Sainsbury, Steve Schwartz, Bill Throop, John Tienson, Mark Timmons, Michael Tye, and Timothy Williamson. I presented portions of this paper in a talk at the University of the Witwatersrand in 1991, and I thank that audience for their comments.

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